Working the Form: George Spencer-Brown and the Mark of Distinction*

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A new sign

At issue here is to advocate for a rare event – the introduction of a new symbol. We are all familiar with symbols such as "+", "-", "&", "%", "§", "©", "@", "⚷", "√", "♂", "♀" and even, pinnacle of the mysterious, the sign for equals, "=". We calculate with them, routinely follow their commands and employ them wherever we might use them. Yet only a minute's reflection suffices and we start to hesitate. Where do we know these signs from and for how long have we known what they mean? How often have we witnessed them, following them but without considering, not even for a minute, their meaning or their distinction? And while we're at it, what is the story with other puzzling signs such as "1", "2", "3" or "a", "b", "c"? What do we do when we read "and"? And what is different when we read "or"? Why is it that still today the truth tables of logic have no entry for "yes, but"? And we could pursue this questioning and would not have even left the pertinent circle of a European or even western symbolic universe.

In fact, however, I do not want to put our symbolic world into doubt, but instead to supplement it with another symbol, one that I would hope that some day we come to use as routinely and unreflectively as the above-mentioned signs. It is a symbol that British mathematician George Spencer-Brown introduced in the year of 1969, leaving the expert world rather unimpressed, in order to take a further step toward the old dream of reducing mathematical calculus to a single sign. George Boole, in his *Laws of Thought,* had reduced the entirety of algebra in the year of 1845 to the two numerals of "0" (for Nothing) and "1" (for Universe), so why should this not be taken further.

Spencer-Brown introduced the sign

$$\neg \quad .$$

Some say it comprises the horizontal bar as sign for negation (~) and the vertical Sheffer *stroke*, |, as a sign for *neither-nor* (Sheffer 1913). But that is a reference for specialists. What

interests me is that Spencer-Brown's so-called *mark of distinction,* or *cross,* contains an injunction that is highly unusual for western thinking, namely to read *a difference as a connection* (or *nexus,* to quote Whitehead 1929, pp. 18ff). For example, to write

$$\overline{a|}$$,

is tantamount to saying: observe the mark of "a" in the context of its distinction from something indeterminate, basically from all that is "not a". This command is already graphically performed: I "see" that this "a" finds itself in this specific place through a distinction, and my gaze wanders involuntarily to the outside of this distinction, which is unmarked here, and back again to the *mark of distinction*, which itself now obtains an operational character, the character of performing a separation, the drawing of a distinction. I suddenly "see" that the distinction of space above and below, right and left, frays and is given a boundary only by a sheet of paper, which is itself only a distinction in a fraying space. And I "see" that this fraying space also pervades the very position at which the "a" had just so unshakeably asserted itself. How can the "a" remain in the place that it does? Who is responsible for this *cross* marking an "a" in distinction to indeterminate otherness? Am I not the one who holds this "a" right in the place in which I find it? One thinks of Italo Calvino's short story "Un segno nello spazio" ("A sign in space"), published in his *Cosmicomiche* in 1965—four years before the publication of *Laws of Form*—and in which the protagonist Qfwfq tries to make himself noticeable with the help of a sign in an initially signless world, which is however quickly overrun with far too many other signs (Calvino 1965).

I would like to support Spencer-Brown in his intention, if it was indeed his, to add this sign, the *mark of distinction,* to our trove of signs. I hold it in all its simplicity to be hugely helpful for artists and art lovers, for scientists and intellectuals, for politicians and priests, for entrepreneurs and speculators, for doctors and patients, and for parents and children to track down their own signs, or to give signs, which otherwise would be overlooked, a frame and thereby a cipher. I am advocating this because this framing can only be had at the price of destabilizing them, and because this destabilization makes us responsible for actions (for example, that of framing), which responsibility we otherwise all too gladly deny.

Spencer-Brown's Symbol can be interpreted in many different ways. And only one interpretation is contained in the practice of logic, as we know it. But already in this context of logic, which Spencer-Brown elaborates on in an appendix to his calculus, is thoroughly deserving of interest. For logically interpreted, the sign

$$\overline{a}\,\vert\, b$$

,

is only to be read as "not-*a* implies *b*," whereby the implication of *b* cannot be obtained without the *a*, so that the negation of *a* manifests itself as an indication of *a* via a detour through a *b*. This is also how Ludwig Wittgenstein presents it: "But it is important that the signs 'p' and '~p' can say the same thing. For it shows that nothing in reality corresponds to the sign '~'" (Wittgenstein 1921, #4.0621).

Let us approach with caution this interesting sign, the mark of distinction. First up, I present the author, George Spencer-Brown, then I proceed to his problem with imaginary numbers, present both of his laws of form, and lastly introduce the key figure of this exercise, the observer. Whoever works with this new symbol is able to understand and describe identities as oscillations. And only this delivers us from a categorically gridlocked world, opening onto a calculus of identities in networks.

George Spencer-Brown

George Spencer Brown was born in Lincolnshire, England, on April 2, 1923. At the age of three his father read him Euclid's *Elements*. After this education, his father sent him to school at the age of six and he never forgave his father for it. Since it took him many years to unlearn what he had been taught there. Already at the age of four George would destroy a spider's web every evening, a web that hung in a shrub in front of his window. Bright and early he would get up to observe how the spider again spun its web anew. Every morning he came too late. The web was always long rebuilt. He wanted to know how this spider was able to spin a horizontal thread. By then he already sort of knew, as he later recounted, that one must destroy a form if one wants to find out how it was accomplished.

After school and after service with the Royal Navy as a telegraphist and as a radio mechanic, as well as some experiments using hypnosis during dental treatment, he entered Trinity College in Cambridge after World War II, where he was a member of the chess, football, and tennis teams, and joined the Cambridge University Club for gliding, winning a few records on the way. He served as a reserve officer for the Royal Air Force. He participated in acrobatic flying competitions, as well as in a theatre group performing plays by Shakespeare, and in 1950/51 worked with Ludwig Wittgenstein on the fundamentals of philosophy. In the early 1950s his interests bore on philosophy, psychology, pedagogy, and the paranormal.

Then he changed to Oxford University, where he began to publish on questions of zoology and comparative anatomy, mathematics, and psychology, and lastly on the role of statistical research in psychology. He wrote his PhD on the dependency of scientific inference on probability (Spencer-Brown 1957; see Martin 2015). In it, he proves among other things that chance presupposes intention, the intention of an observer to accept something as chance, because it accords with the pattern of that which is without pattern.

In the 1960s he began to work as the Chief Logic Designer at Mullard Equipment, then developed together with his (invented and soon to be disinvented) brother David J. Spencer Brown elevator control units, as well as, for British Rail, transistor counting modules which employ complex arithmetical signals. During these same years he worked with Bertrand Russell on the foundations of mathematics, and apparently was then treated by Ronald D. Laing, a psychiatrist influenced by the existentialist Jean-Paul Sartre, with whom, Laing, I mean, he ended up working. He went into psychotherapy in order to increase sporting performance through hypnosis and sleep learning techniques, and trained and educated gifted and highly-gifted children. Laing's book *Knots* (1970) about human relations tending to get entangled in knots appears to owe much to Spencer-Brown.

Some years ago the first volume of Spencer-Brown's autobiography, titled *Autobiography 1: Infancy and Childhood* (Spencer-Brown 2004), of which further volumes were to follow, was published, in which he details his childhood and teenage years, and his hatred toward his mother. Sometime in the 1990s Spencer-Brown, to the confusion of the librarians, who had hitherto filed him under Brown, added a hyphen to his surnames.

Imaginary numbers

In 1969, Spencer-Brown's books, *Laws of Form* finally appeared, in which he attempts to make mathematical logic familiar with possibilities that he and his brother had successfully employed in the engineering sciences. In so doing, the particular issue at stake involves calculating with the imaginary number i, the mathematical result of the equation

$$x^2 + 1 = 0,$$

thus

$$x^2 = -1.$$

This equation is not unambiguously resolvable either with the squaring of the principal root,

$$\sqrt{x} = \sqrt{-1},$$

or by division,

$$x = -1/x.$$

$\sqrt{-1}$ is neither +1 nor -1, and thus somehow is simultaneously both +1 and -1. And

$$x = -1/x$$

leads to a self-referential statement that had been prohibited since Bertrand Russell and Alfred Whitehead North's *Principia Mathematica* (1910), a statement leading to contradiction: x is -1 divided by itself, x.

The solution to these types of equations consists in the introduction of imaginary numbers, which are thus defined as this solution, and which ever since Leonhard Euler the letter *i* has been used to indicated. Through the number *i,* such equations add to natural, whole, rational, irrational and real numbers to make complex numbers.

The intuition that Spencer-Brown pursues in the *Laws of Form* is that the introduction of imaginary numbers makes all numbers complex numbers, not only the newly added ones. But what is a complex number? In John Stillwell's history of mathematics (Stillwell 2002, pp. 383f) we are told that complex numbers, even when not given under this name, were discovered already by Diophantos of Alexandria supposedly around 250 A.D., as he formulated his arithmetic of pairs. Ever since, "complex" is used to refer to the unity of a multiplicity, which can be reduced neither to a simple unity nor to a sheer multiplicity. Complex numbers, spaces, and functions consist in at least a pair of two unities, which presuppose one another, and cannot be reduced either one to the other.

Let us remark in parentheses that Jurij M. Lotman, in his book *Universe of the Mind*, rediscovered such pairs for semiotics and referred to them therein as tropes (Lotman 2001). Nathaniel Hellerstein, with recourse to Spencer-Brown's *Laws of Form,* developed a so-called *Diamond Logic* (Hellerstein 1997), a non-Aristotelian logic in which the classical truth values of *true* and *false* are supplemented with the two values of *true, but false* and *false, but true* (which oscillate with a *unit delay*). Alfred Korzybski (1933) had similar intentions of pursuing a non-Aristotelian logic, the point of which was, as in Hegel's Logic, in Karl Marx's theory of capitalism, or in Maurice Merleau-Ponty's theory of perception, not the problem of describing the results of processes as either true or false but instead that of describing these processes themselves. This has little to do with efforts in paraconsistent or fuzzy logic, because it is not a matter here of the introduction of ontically indeterminate values but instead

of processually operational ones. Hegel spoke of *Übergänge* (transitions). Continuing on from Hegel and from Martin Heidegger is Gotthard Günther's (1979) attempt to construct a polyvalent logic, one able to describe not only what is in negative and positive terms, but also to understand which stories of reflection in the medium of negation it stems from. Stories of reflection are stories going back and forth in checking on the plausibility, validity, and sincerity of a claim; and a medium of negation is a loosely coupled assemblage of claims and denials, or acceptations and rejections, as Günther prefers to call them, prepared to do such a checking.

Laws of Form

The problem that Spencer-Brown ties to solve in *Laws of Form* is the description of a unity that consists irreducibly in a duality (Glanville 1982). And the stimulus that he brought from the electronics of circuitry into mathematical logic consists in thinking of this unity not substantially, as with the essence of the Greeks, nor relationally, as a relation between things or elements, but instead operatively, as calculus, as arithmetic procedure.

Whence the famous first two sentences of the first chapter, whose title reads "The form" (Spencer-Brown 1969, p. 1):

> *We take as given the idea of distinction and the idea of indication, and that we*
> *cannot make an indication without drawing a distinction. We take, therefore,*
> *the form of distinction for the form.*

The *one* beginning consists in *two* ideas, the idea of distinction and the idea of indication. When we want to indicate something, we have already distinguished something. The form is the form of the distinction.

Then we read this emphasized as a definition (ibid.):

> *Distinction is perfect continence.*

A distinction is perfect continence in the sense of perfect inclusion. A distinction contains the world and brings this world forth, because it can only be made in this world, if it indicates something in this world and thereby distinguishes the indicated something from the world. The distinction thus includes both the indicated thing and that from which it is distinguished. In addition it includes itself, the distinction, as separation between these two sides. Without the distinction being encountered, actually accomplished, i.e. made operative, there can be

nothing to say about that which it indicates and that from which the indicated thing is distinguished. Under the concept of form all this becomes observable, whereby even the indication of the form of the distinction includes an indication and a distinction, for which the inverse also applies.

Not least the concept of form thus also contains the observer that makes the distinction, because without this observer no distinction would be made. This is also what is meant when distinctions are understood not categorically as the supposed objective order of an already classified world, but instead constructively or onto-genetically, as Heinz von Foerster (1993) puts it, as operations-of-an-observer, one able to distinguish thus and otherwise, but who first of all must make a distinction in the first place in order to be able to find himself again at some point within his own distinctions, and thus to turn himself into the object of his own observations. For Heinz von Foerster, just as for Niklas Luhmann (and for Johann Gottlieb Fichte and Jacques Lacan) it was important to stress that the distinction that the observer applies in order to observe himself cannot as far as he is concerned be observed in the moment in which he applies it. The observer who observes himself is thereby his own blind spot. He cannot observe himself. He can only observe something with which he mistakes himself.

It is not possible to grasp a complexity, a multiplicity as unity, namely a multiplicity of aspects as unity of an operation, in a more terse or reduced fashion. And of course this understanding of complexity presupposes that one acknowledges a paradox as well as a self-reference—the paradox of unity as multiplicity and the self-reference of distinction to itself. To carry out an operation means to make use of what it is that the operation should give rise to. And this means to refer to the operation within the operation, which is carried out as the operation.

The Observer

Out of this results a concept of form that, as Niklas Luhmann (1999) had understood, on the one hand no longer depends on the opposite concept of matter, as with Aristotle, nor on that of content, as with eighteenth-century aesthetics, but rather indicates itself without opposite and develops out of itself all possible opposites, given the corresponding distinctions. And out of this comes a concept of form that is dependent on an observer, if by observer one understands one who draws a distinction. This observer is not tied to having to open his eyes. A distinction can be of a visual, auditory, tactile, olfactory, emotional or rational type. Form,

distinction, and observer are the most general concepts of any possible theory of cognition, of which organisms are capable but also mental or social operations.

Both features together—the self-reference of form and observer-dependency of the form, such that the first distinction and the observer are, as Spencer-Brown formulates it, identical "in the form"—establish the attractiveness of the calculus of form first for systems theory and additionally for all cognitive theory, if by cognition one understands the giving rise to something, cognition itself included, through the making of a distinction. That is why the calculus of form delivers the basic text for all types of cognitive systems, whether they be organic, physical, social, or artificial. And that is why the calculus of form demands a new formulation of the concept of system, which no longer refers to relations between elements much less to a coherence of elements secured through some sort of order or reason of things, but instead to the recursivity of operations, which draw a distinction, and therewith also a boundary, and thus make both sides of the distinction observable, which is to say, include the excluded as excluded.

The "complex" is henceforth that which oscillates and which reproduces itself in oscillation. The challenge of the calculus of form put forward by Spencer-Brown consists in seeing that there is nothing that is simpler than this. Two axioms are enough in order to establish a calculus with which this insight is born out. These two axioms define the two laws that give the book its title. The first axiom defines the *law of calling* (1969, p. 2):

> *The value of a call made again is the value of the call.*

If one makes a distinction and then makes it again, this changes nothing in the value of the distinction, even if with the repetition one becomes manifest as observer and thereby introduces a new value. The second axiom defines the *law of crossing* (ibid.):

> *The value of a crossing made again is not the value of the crossing.*

If one draws a distinction in the sense of crossing from the outside of the distinction to the inside of the distinction, and one subsequently crosses again to the outside, then one stands there empty-handed, or more technically expressed: one lands in *unmarked space*. Both these laws fix unity as the unity of its repetition and the void as the sublation of a unity.

Spencer-Brown required no more than these two laws in order to develop a calculus of form—he speaks of a *calculus of indications*—wherein it is shown how it is possible to calculate with distinctions that are nested in one another. Every form, no matter how manifold, can be reduced either to the marked state, that of a distinction, or to the unmarked

state, that of the void. Critics of this calculus of form such as Paul Cull and William Frank (1979) have thus objected that Spencer-Brown achieved no more than a reformulation of Boolean algebra, which as for it makes do with two numbers, 1 for Universe and 0 for Nothing. Defenders of this calculus, such as the mathematician Louis H. Kauffman (1987), point out that, on the contrary, Spencer-Brown supplements Boolean algebra with the insights of Charles Sanders Peirce, whose semiotics show that one does not understand signs of any kind, and is much less able to employ them, if one does not take into consideration a third, an interpretant, for whom the sign makes sense.

Identity as Oscillation

Spencer-Brown develops his calculus of form to the point at which a distinction can be indicated and distinguished purely arithmetically. In mathematics this point is the introduction of imaginary numbers, and in logic, that of the sublation of the Aristotelian laws of identity,

$$A = A,$$

of non-contradiction,

$$\neg (A \wedge \neg A),$$

and of excluded middle,

$$A \vee \neg A.$$

In the place of this Aristotelian logic we have the insight according to which identity can only come in the frame of a negation, which is to be understood as implication, even as oscillation (see also Baecker 2013):

$$a = \overline{a\rceil}\rceil \quad .$$

An *a* is only an *a* (identity), if it is distinguished from an outside, which it is not (negation), whose existence it nonetheless presupposes as an outside to the distinction (implication).

Instead of the Aristotelian laws of identity, non-contradiction, and excluded middle, we have the cybernetic laws of paradox,

$$a \neq a,$$

of ambivalence,

$$a \wedge \neg a$$

and of control,

$$a \vee a.$$

Organization theorist Philip G. Herbst (1976) was one of the first to investigate the supposition that Spencer-Brown's form of distinction offers the possibility to start without presuppositions and both to reign in all presuppositions in the course of the process as well as to modify them. In this way he believed to have avoided Platonism, or the belief in eternal ideas, as well as positivism, or the belief in already existing "facts" given as "data," and Kantianism, or the belief in transcendentally grounded principles of knowledge (space and time), and therefore to be able to analyse what it is that industry, bureaucracy, and management really mean. Niklas Luhmann (2013/14) continued on with this program at the level of a theory of society.

    Spencer-Brown's calculus of form consists in developing arithmetic with distinctions from a first instruction—"draw a distinction"—to the re-introduction ("re-entry") of the distinction into the form of distinction, in order to be able to show in this way that the apparently simple, but actually already complex beginning involved in making a distinction,

$$\daleth \quad ,$$

can only take place in a space in which the distinction is for its part introduced again,

$$\daleth \quad .$$
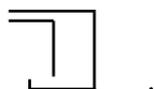
The observer who makes this distinction through it becomes aware of the distinction, to which he is himself indebted. That is why Heinz von Foerster, the founder of second-order cybernetics (see von Foerster 2003), reacted so enthusiastically to the calculus of form and wrote the first review of it for the *Whole Earth Catalog* in spring 1969. Distinctions have their mathematical and logical place at the level of second-order observation. They are what they are, namely the distinction made by an observer, only if they are observed by a further observer as this distinction, namely in view of their form. This further observer can be the

first, self-observing observer. However he can also be another observer, who then appears as a socius, as a co-partner of the first observers, and with him together grounds a not-necessarily peaceful society.

Spencer-Brown's calculus of form is developed to the point at which it can be shown that this reintroduction, and with it self-observation and socialization, is possible but nonetheless has its price. This price consists in the fact that, in the form of their distinction, reintroduced distinctions cannot be led back to either a simple distinction, or to the void. Instead they oscillate imaginarily, and thereby indissolubly, between both possibilities.

That is, however, precisely the characteristic of a complex unity. It is both something and nothing at once, and obtains its reality from the oscillation between both these possibilities. This is the point from which every science of cognition begins (see for example Varela 1988). Spencer-Brown, however, subsequently only stresses in his text that from this point one could one day develop the calculus to be able, for example, to understand what it means to count and to remember.

George Spencer-Brown's calculus has received a scattered reception in the domains of therapy, sociology, and mathematics (see Baecker 1999 and 2013). Whoever wants to reckon with empty spaces, with self-reference, and with the inclusion of the observer in his observations, cannot avoid this calculus and its ideas.
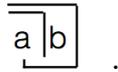
An exercise

An exercise with which you can make yourself familiar with the *mark of distinction* symbol is the simplest of all. Take an object of your interest, some *a*, frame it by a distinction that you'd made long ago but without having been aware of it:

$$\overline{a|} \quad ,$$

then ask what it is that you could have excluded – some *b* – by which you had made this *a* into a focus of interest:
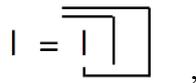
$$\overline{a}\,|\,b \quad ,$$

and grasp that the excluded *b* is as important for the identity of *a* as the marking of *a* as distinguished from *b*:

$$\overline{a\,|b|}\,.$$

Repeat this exercise with small and large, important and unimportant, non-living and living things. They all get caught in oscillation and betray to us something about their existence.

Give yourself, *I*, such a form:

$$I = \overline{I\,|\,|}\,,$$

and experiment with what you thereby are exposed to see.

> *Draw a distinction.*
> *Watch its form.*
> *Work its unrest.*
> *Know your ignorance.*

Translated by Steven Corcoran.

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